**Introduction**

Recently, as a result of the FFR hikes, it has caused great turbulence in the global financial markets. This not only brough about a short-term strength in the US dollar, which significantly influenced the FX market, but also affected the pricing of the rest of the assets as well as the anchoring of sovereign credits. Many investors focused on the volatility of the FFR, the tone given by the Feds, and the official dot plots, in an attempt to make better predictions about the FFR to protect their portfolios or even make profits. Notably, Taylor Rule is a popular approach for Fed to build the relationship between the FFR and the macro conditions, which are explained mainly by Resource Gap measured by GDP and Inflation measured by PCE. Based on the Taylor Rule, the investors and institutions can make projections for the FFR with the publicly available information.

According to the basic assumptions of time series modelling, forecasts at moment t cannot use future information, but only current moment as well as ever information [17]. However, in the actual publication of economics data, although different economic indicators reflect economic conditions over the same period of time, they cannot be regarded as variables with the same time stamp due to the difference in publication time. This is also the case in this study, although based on the Taylor Rule, the FFR can be calculated using the relevant macroeconomic indicators for this period, the corresponding macroeconomic indicators are published 1-2 days after the publication time of the FFR, and therefore in practice it is not possible to use such future information for FFR determination. It can be visualized through Fig.1, that the FFR is released at the end of the quarter, but the corresponding macro data are released afterwards. In reality, researchers generally use the previous period's macroeconomic data as the input factor for the Taylor Rule, however, this can lead to a lagged effect as the calculation of the FFR ignores the most recent period's economic situation. There are also many relevant financial institutions that present alternative data for use, but this is not always publicly available and authoritative.

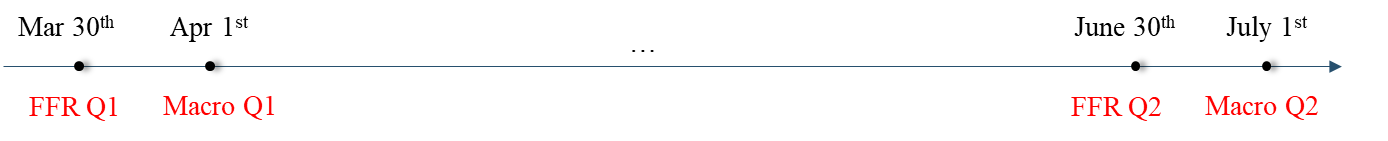


Figure 1. The release example of FFR and Macro Data

Moreover, especially the U.S. Gross Domestic Product (GDP) lags much more compared to the inflation data, as it has multiple revisions to provide more reliable predictions. It is because the GDP revisions involve updating and refining previously released GDP data to provide a more accurate and comprehensive representation of economic activity. Revisions are a normal part of the economic data reporting process, as initial estimates are often based on incomplete information and preliminary data. 这里加上Figure 2的描述，以及你填上Figure 2的名字。然后说adjustment的细节在Appendix提到，要提到的就是你ppt里面做的那一页很多字的东西，记得附上reference。

A screenshot of a graph

Description automatically generated

Figure 2.

Although Taylor Rule has the obstacles mentioned above, it also proves the effectiveness of the Resource Gap and Inflation towards the FFR prediction, and the indicated form is linear, which can be further studied by applying the rolling linear regression to make more accurate estimation as the coefficient are ever-changing instead of setting preliminarily. In this study, we aim to embed the information from daily Treasury Yield data, which is achieved by applying the Hidden Markov Model with Gaussian Mixture Models (GMM-HMM), to make more recent information projection. And by combing this obtained information and the original Macro data, a rolling regression model is fitted with the historical data, and hence makes a one step ahead prediction for the FFR rates.

**Literature Review**

1. Federal Reserve Rate
   1. Monetary Policy

The Federal Reserve, through its monetary policy adjustments, creates a favorable economic environment characterized by appropriate employment rate and stable price [1]. When the aggregate demand lags the economy's capacity to produce, it results in increased unemployment rate and reduced inflation. To counter this, the Federal Open Market Committee (FOMC) intervenes by reducing interest rates and implementing an expansionary monetary policy to stimulate aggregate demand, thereby helping stabilize the economy.

Conversely, if demand for goods and services becomes excessively strong, it can lead to unsustainably employment rate and increased inflation, leading the Federal Reserve employs a contractionary monetary policy by elevating interest rates to guide economic activity back to normal level. The procedure through which the FOMC enacts expansionary and contractionary monetary policies to achieve its goals can be summarized as shown in Fig.1 [2].

A diagram of a monetary policy

Description automatically generated

Figure 3. Federal Reserve Monetary Policy

* 1. FFR

The primary method to exert monetary policy is the adjustment of the federal funds rate (FFR) [3]. Banks maintain reserve balances at the Federal Reserve to fulfill unforeseen liquidity requirements, so they engage in borrowing and lending of reserves among one another based on their specific needs. The federal funds rate represents the interest rate at which banks engage in overnight borrowing, which plays a pivotal role in determining the expense of short-term credit.

To impact the federal funds rate, the FOMC can modify the interest rate applied to bank reserves. This adjustment leads to changes in the federal funds rate, aligning it with the FOMC's desired objectives and influencing the cost of short-term interbank credit.

In response to the 2008 economic crisis and subsequent economic recession, Federal Open Market Committee lower the target for the federal funds rate from 5.25% in mid-September 2007 to near zero by the end of December 2008 (See Fig. 2) [4]. This rate reduction was part of the Fed's strategy to stimulate economic activity and provide liquidity to the financial system. The goal was to make borrowing cheaper for banks, businesses, and consumers to encourage spending, investment, and lending. Together with various monetary policies such as buying back government securities, the market responded by purchasing large-scale asset, consequently fostering economic growth, job generation, and a gradual resurgence of inflation toward 2% [2]. Notably, in December 2015, the Federal Open Market Committee initiated the process of increasing the target for the federal funds rate, transitioning from its near-zero level to a more conventional rate. Subsequently, in October 2017, the Federal Open Market Committee embarked on the gradual reduction of its securities holdings, marking another substantial step toward the normalization of monetary policy [5]. As part of this shift, the Committee conveyed that future adjustments in the federal funds rate would serve as the primary mechanism for altering the overall stance of monetary policy.

In the recent context, the U.S. economy has been experiencing a robust recovery after a period of economic disruption, possibly due to the COVID-19 pandemic. Annual inflation rates have risen above the Federal Reserve's target of 2%. Inflation, as measured by the Consumer Price Index (CPI), is at 3.5%, and core inflation (excluding food and energy) is at 2.8%. To address these economic conditions, the Federal Reserve announces an increase in the target FFR from 0.25% to 0.50% [5]. This is the first-rate hike in several years, signaling the central bank's confidence in the strength of the economic recovery.

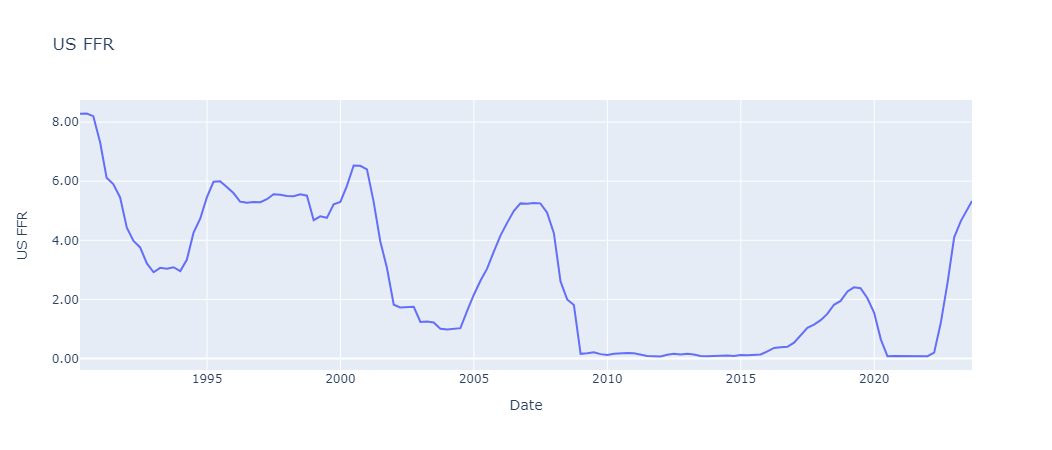
**

Figure 4. US Effective FFR Diagram

1. Taylor Rule

In a paper published in 1993, John Taylor showed how monetary policy in the United States from 1987 to 1992 was approximated by a formula that related the federal funds rate to three variables. The first variable is the inflation-adjusted long-run federal funds rate, the second is the deviation of current inflation from the 2% target set by the Federal Open Market Committee (FOMC), and the third is the percentage difference between actual GDP and its potential level.

Taylor Rule takes the following general form, with the specific meanings of the indicators shown in the table below [6].

|  |  |
| --- | --- |
| Variable | Implication |
|  | Federal fund Rate |
|  | Real Neutral Rate |
|  | Expected Inflation |
| 𝜋∗ | Target Inflation |
| − | Percent deviation between the current real GDP and the long-term linear trend in GDP |

Table 1. Variables Explanation in Taylor Rule

The Taylor formula illustrates that when inflation surpasses the 2% target, the federal funds rate increases at a rate 1.5 times that of the inflation increase. Furthermore, if the GDP exceeds its potential level, the federal funds rate increases by 0.5 times the difference between the GDP and its potential level.

The Taylor rule embodies the fundamental principles of monetary policy discussed earlier. Firstly, when the real long-term neutral federal funds rate, the actual and target inflation rates, and the real GDP level and its potential are all known, the adjustment based on the difference between the GDP and its potential level is zero, making FFR prediction feasible. Secondly, it advocates for higher FFR in response to rising inflation or increased resource utilization, and lower FFR when inflation subsides or resource utilization declines. This alignment corresponds with the Federal Reserve's dual mandate. Lastly, the equation dictates that the federal funds rate should be adjusted by more than a one-to-one ratio when inflation experiences upward or downward movements, a characteristic often referred to as the Taylor principle.

Federal Reserve officials and economists later introduced several alterations to the variables used in the Taylor Rule, aiming to provide a more accurate representation and interpretation of shifts in the real-world scenario and policy structure. As a result, numerous revised iterations emerged (Table. 2) [7].

|  |  |  |  |
| --- | --- | --- | --- |
| Rules | Formula | Coefficient of Output gap | Coefficient of Inflation gap |
| Bernanke Rule |  | 1 | 0.5 |
| Evans Rule |  | 2 | 0.5 |
| Yellen Rule |  | 0.5 | 2 |
| Bullard Rule |  | 0.1 | 1.5 |

Table 2. Adjusted versions of Taylor Rule

1. Treasury Yield

U.S. Treasury yield is the yield on U.S. government bonds, whose metric measures the return an investor can earn by purchasing U.S. government bonds. U.S. government bonds are bonds issued by the government to raise funds and are usually classified as having different maturities, including short-term, intermediate-term, and long-term bonds.

Treasury yield is often used by investors and economic observers as an indicator of risk and market expectations. Based on the risk-neutral interpretation, treasury yields are equal to the average value of expected future short rates [8]. A low Treasury yield may indicate market concerns about future economic uncertainty, while a high Treasury yield may reflect investor optimism about economic growth and inflation. In addition, Treasury yield is used to determine the pricing of other financial instruments, such as mortgage rates and corporate bonds. Treasury yields can reflect economic conditions, monetary and fiscal policies, and expectations about future economic activity, real interest rates, and inflation [9]. What can be agreed upon is that whenever macroeconomic data is released differently than the consensus, treasury yields always have a noticeable jump, indicating the influence of macro economy situations to the treasury yields. In this research, we take 6 U.S. treasury yields from Bloomberg into consideration according to the dataset coverage [10], and the remained NAN values are forward filled based on the previous dates’ yield data. A visualization of the Treasury Yields is shown in Fig. 3:

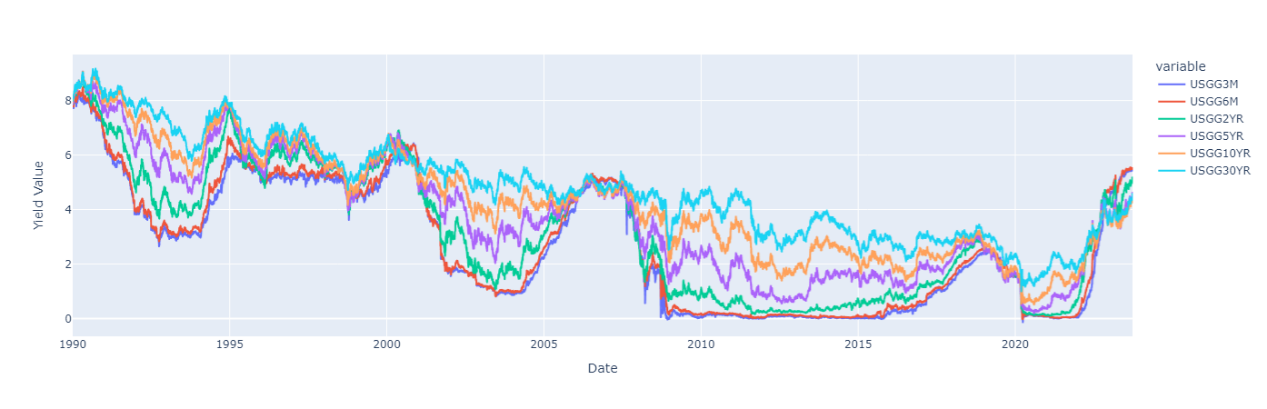


Figure 5. US Treasury Yields Diagram (Maturities including 3, 6 Months, 2, 5, 10, and 30 Year)

Additionally, the term structure of the treasury yields is also an important dimension to understand the economic situation at that point of time. First, regarding the shape of the yield curve at a given point in time, broadly speaking the curve should show an upward sloping trend, but the rate of increase in interest rates decays as maturity increases. This shape is supported by the principles of Expectation, Bond Risk Premiums, and Convexity Bias as stated by Ilmanen. [11] In the history, researchers were interested in proposing models to fit the treasury yield curve better or analyzing the components from the yield curve, and there are indeed the components named level, slope, and curvature that is economically explainable to the structure of yield curve [12]. By reviewing the yield curve in Fig. 3, we can visualize that the up and down shifts in the interest rate curve are essentially joint (as measured by level), and that in most cases the curve with high maturity lies on top (can be illustrated by slope). Various experimented models can prove the existence and significance of these three components, such as the Nelson-Siegel Model [13, 14]. However, this model required some essential tricks to determine the parameters in the model so as to provide a better simulation result. There is another easier method, which is named the Principal Component Analysis that is frequently used in Statistics and Data Science to reduce the dimensionally of the features and get the important ones, that is proved to be useful to model these three factors [15]. From this method, the first three important factors are representing the level, slope, and curvature components in the yield curve, while the remaining are assumed to be noise and filtered out.

Indeed, treasury yields have strong correlation with FFR, especially in the short-term tenor parts, which is because the short-term rate is linked to the FFR to some extent. [16] However, the raise of FFR doesn’t necessarily provide evidence for the change of long-term yield. It can be concluded that this kind of situation could flatten the yield curve as the short side increase more than the long side. We perform a simple correlation measurement to the FFR and the monthly treasury yield rate in Fig. 4, and it is found that the correlation is very close to 1. Whereas, if we lag the treasury yield rate 1 year before and still apply the correlation test, the relationship is still relatively stable. It seems that the treasury yield cannot be directly used as an instrument to project the FFR in the future.

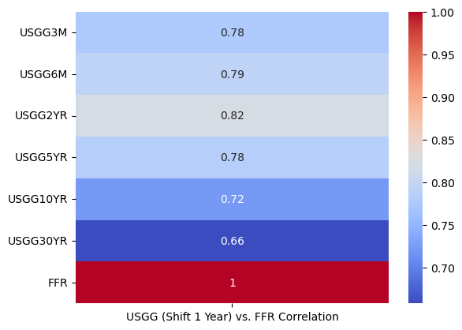
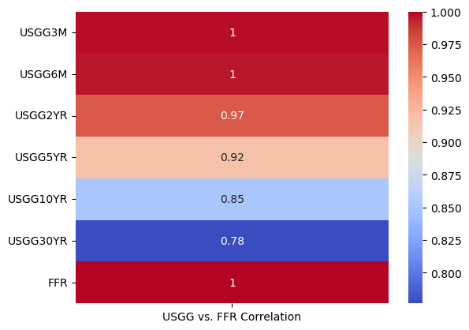


Figure 6. Correlation between the FFR and monthly Treasury yield (Left) v.s. Correlation between the FFR and monthly Treasury yield on year before (Right)

**Research Objective**

1. Transform the treasury yields data into explanatory features, and then feed them to the GMM-HMM to obtain the leading macroeconomics factors.
2. Construct the model based on the traditional Taylor Rule and generated new macro factors.
3. Adjust the incorporated information window to test the projection performance of the model.

**Dataset Description**

Both the Treasury yield data and Federal Funds Rate are downloaded from HKU Bloomberg Laboratory [10]. The Macro data of Real GDP, Potential GDP, and PCE are downloaded from FRED Economic Data website [18]. The Resource GAP is measured by the logarithmic transformation of Real GDP minus the logarithmic transformation of Potential GDP. And all the data are set to start from 1990-01-01.

**Methodologies**

1. Principal Component Analysis (PCA)

The goal of the PCA method is to reduce the dimensionality of the data without losing much meaningful information, which can even filter out the noise. In this study, the PCA is utilized to reduce the components of yield curve from 6 original curves to 3 meaningful components that can be classified to Level, Slope, and Curvature. The detailed steps are shown below:

1. Get the covariance matrix of all yields data .
2. Calculate the eigenvalues and eigenvectors of .
3. Select some best eigenvalues, and project the original data to the corresponding eigenvectors. The eigenvectors represent the new dimensions that the original features are projected to, which are called the principal components. And the eigenvalues are the mode of the new projected vectors on the principal components.

From these three steps, we can obtain the 3 components on a daily basis, and they can also be combined to rebuild the yield curve. An example of the PCA method utilized for 1991 Q1 is shown in Fig. 5. The left diagram shows the cases of Level, Slope, and Curvature, which can be easily understood. The Level value for yield with different maturities are all greater than 0, meaning that the curve can move upwards or downwards simultaneously according to the change of Level.

A graph with lines and numbers

Description automatically generated A graph with different colored lines

Description automatically generated A graph with blue and orange lines

Description automatically generated

Figure 7. The values that the original 6 yield curves projected on the principal components (Left); The daily Level, Slope, and Curvature values provided by PCA during 1991 Q1 (Middle); Rebuilt Yield Values compared to Original Yield Values of USGG6M (Right)

1. Hidden Markov Model with Gaussian Mixture Models (GMM-HMM)

Under the assumption, macroeconomic conditions are affected by the newly released FFR, after which it continues to operate on its own. The yield curve is to some extent a reaction to the macro data, and when there are optimistic expectations about the economic trend, the yield curve will go down. The state of the macroeconomic situation is positional during this period of time, but we can observe changes in the Treasury. And the change in Treasury yield is not only affected by GDP and Inflation, but it may also be affected by other macro data releases (e.g., non-farm payroll), Fed speeches, changes in the political situation, and all of these are likewise outward forms of the macroeconomic situation. Therefore, this study aims to use the GMM-HMM to create new variables from the Treasury yield data that can describe the macroeconomic situation from another aspect.

For the HMM, we had the notations:

Set of hidden states:

Set of observable states:

The series of hidden states:

The series of observable states:

Parameters:

Hidden States Transition Probability (how the hidden states from change to ):

How the hidden states can come up with the observable states:

The initial states at , which is the probability

It is assumed that the series of hidden states follows Markov Assumption, that the current state at is only dependent on the state at and the constant hidden states transition probability matrix , but not any of the prior state data. And the observable states at is only determined by the hidden states at time and the matrix .

The above statements are for the general discrete HMM model, but in this study, the states are not discrete, but instead continuous. Hence, the discrete probability measurement cannot be directly used in this situation, and Gaussian Mixture Model can be applied instead. Gaussian Mixture Model is a probabilistic model used for representing a mixture of multiple Gaussian distributions, which is an extension of the traditional Gaussian Model. It is a generative model that assumes all the data points are generated from a mixture of several Gaussian distributions with unknown parameters. It can be formulated step-by-step, and the final Gaussian Mixture Model is applied to substitute the probabilities in the discrete HMM model:

P.D.F. of Univariate Gaussian Model:

P.D.F. of Multivariate Gaussian Distribution:

P.D.F. of Gaussian Mixture Model:

where:

is the weight of the k-th component, with

is the P.D.F. of the k-th Multivariate Gaussian Distribution

The target of this study is to obtain the last projection of the hidden states, which is . To obtain the estimation of this value, the Viterbi Algorithm is utilized, which is detailed in the Appendix.

1. Rolling Regression

**Results Analysis**

1. Traditional Taylor Rule

**Conclusion**

**Appendix**

1. **The U.S. GDP Revision Details**

Details放在这里

1. **Definition of Effective Federal Funds Rate**

The federal funds market consists of domestic unsecured borrowings in U.S. dollars by depository institutions from other depository institutions and certain other entities, primarily government-sponsored enterprises. The effective federal funds rate (EFFR) is calculated as a volume-weighted median of overnight federal funds transactions reported in the FR 2420 Report of Selected Money Market Rates [].

1. **Viterbi Algorithm Details for solving GMM-HMM**

**Reference**

[1] Hayes, A. (n.d.). Federal Reserve System: What it is and how it works. Investopedia. <https://www.investopedia.com/terms/f/federalreservebank.asp>

[2] Monetary policy: What are its goals? how does it work?. Federal Reserve Board - Monetary Policy: What Are Its Goals? How Does It Work? (n.d.). <https://www.federalreserve.gov/monetarypolicy/monetary-policy-what-are-its-goals-how-does-it-work.htm>

[3] How does the Federal Reserve Affect Inflation and employment?. Board of Governors of the Federal Reserve System. (n.d.). <https://www.federalreserve.gov/faqs/money_12856.htm>

[4] Rich, R. (n.d.). The great recession. Federal Reserve History. https://www.federalreservehistory.org/essays/great-recession-of-200709

[5] Current and past monetary policy announcements can be accessed at http://www.federalreserve.gov/monetarypolicy/fomccalendars.htm.

[6] Taylor, John B. "Discretion versus Policy Rules in Practice." Carnegie-Rochester Conference Series on Public Policy, vol. 39, no. 1, 1993, pp. 195-214.

[7] Bernanke B. Why Are Interest Rates So Low. <https://www.brookings.edu/blog/ben-bernanke/2015/03/30/why-are-interest-rates-so-low/> , 2015.

[8] Diebold, Francis X, et al. “Modeling Bond Yields in Finance and Macroeconomics.” *American Economic Review*, vol. 95, no. 2, 1 Apr. 2005, pp. 415–420.

[9] Evans, Charles L., and David A. Marshall. “Economic Determinants of the Nominal Treasury Yield Curve.” *Journal of Monetary Economics*, vol. 54, no. 7, Oct. 2007, pp. 1986–2003.

[10] Bloomberg Terminal Data on Treasury Yields (USGG). Bloomberg Terminal, 2023-09-25.

[11] Antti Ilmanen, and Salomon Brothers. Overview of Forward Rate Analysis. 1995.

[12] Afonso, António, and Manuel M.F. Martins. “Level, Slope, Curvature of the Sovereign Yield Curve, and Fiscal Behaviour.” Journal of Banking & Finance, vol. 36, no. 6, June 2012, pp. 1789–1807.

[13] Nelson, Charles R., and Andrew F. Siegel. “Parsimonious Modeling of Yield Curves.” The Journal of Business, vol. 60, no. 4, 1987, pp. 473–489.

[14] Diebold, Francis X., and Canlin Li. “Forecasting the Term Structure of Government Bond Yields.” Journal of Econometrics, vol. 130, no. 2, Feb. 2006, pp. 337–364, https://doi.org/10.1016/j.jeconom.2005.03.005. Accessed 28 Mar. 2019.

[15] Barber, Joel R., and Mark L. Copper. “Principal Component Analysis of Yield Curve Movements.” Journal of Economics and Finance, vol. 36, no. 3, 11 Aug. 2010, pp. 750–765, https://doi.org/10.1007/s12197-010-9142-y. Accessed 5 Feb. 2023.

[16] Seip, Knut Lehre, and Dan Zhang. “The GDP, the US Treasury Yield and the Federal Funds Rate: Who Follows Whom, When and Why?” *Journal of Financial Economic Policy*, vol. ahead-of-print, no. ahead-of-print, 8 June 2021.

[17] Taylor, Stephen J. *Modelling Financial Time Series*. Hackensack N.J., World Scientific, 2013.

[18] FRED. “FRED.” *Stlouisfed.org*, Federal Reserve Bank of St. Louis, 2022, fred.stlouisfed.org.